

# Titles and Abstracts

Vienna, June 20th-24th

## **Emilio Acerbi**

*Stable periodic arrangements arising from nonlocal energies*

Critical points of equilibrium models based on a free energy functional exhibit various pattern structures. These systems are characterized by the presence of coexisting phases, whose distribution results from the competition between short and long-range interactions. Studying an energy-driven sharp interface model with long-range interaction being governed by a screened Coulomb kernel it is possible to devise number of criteria for the stability of lamellar configurations, to ensure that they are indeed strict local minimizers. We also give a sufficient condition to ensure a nontrivial periodic 2D minimal energy configuration.

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## **Giovanni Alberti**

*The vanishing mass conjecture and its geometric interpretation*

G. Bouchitté formulated the “Vanishing Mass Conjecture” about twenty years ago, in the context of optimization of light elastic structures. Since then the only progress has been obtained by J.F. Babadjian, F. Iurlano and F. Rindler who proved the conjecture in a particular case in 2021. In this talk I will illustrate this conjecture, placing the emphasis on its geometric nature, and explain some connections to other results and open questions in Geometric Measure Theory.

**Luigi Ambrosio**

*On some variational problems involving functions with bounded Hessian*

Motivated by problems related to deep learning, we study variational problems involving bounded Hessian functions, i.e. functions whose gradient is a function of bounded variation. In this context, particularly interesting are density results for piecewise affine functions and questions regarding extremal points. Work in progress with M.Unser, S.Aziznejad, C.Brena.

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**John Ball**

*Axisymmetry of critical points of the Onsager functional*

The talk will discuss critical points of the Onsager functional for liquid crystals, giving in particular a simple proof of their axisymmetry in the case of the Maier-Saupe molecular interaction, a classical result of Fatkullin Slastikov (2005) and Liu, Zhang Zhang (2005). For general molecular interactions the smoothness of critical points is proved, and for a wide class of interactions the existence of non-axisymmetric critical points is established.

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**Kaushik Bhattacharya**

*Soft behavior in hard materials*

Soft behavior is the ability of a material to change macroscopic state with little resistance. In this talk, we discuss two examples of soft behavior due to a combination of non-convexity and spatial randomness. The first example concerns liquid crystal elastomers. The ideal version of the neo-classical Bladon-Terentjev-Warner theory leads to a non-convex energy. Desimone and Dolzmann showed that the relaxation of this energy leads to soft behavior. Remarkably, we show through computation and experimental observations that aspects of this soft behavior persists even in the non-ideal case that is locally convex but has random spatial heterogeneous. The second example concerns the widely used ferroelectric material lead-zirconate-titanate (PZT) which is a solid solution between lead-zirconate and lead-titanate. This material is ferroelectrically hard except at a particular compositions (known as the morphotropic phase boundary), and we show that this is the result of a

competition between chemical randomness and electrostatic ordering. The talk is motivated by the recollection of the result of the Irene Fonseca on the relaxation of the elastic energy of crystalline solids.

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**Guy Bouchitté**

*Optimal design versus maximal Monge-Kantorovich metrics*

A remarkable connection between optimal design and Monge transport was discovered in the years 1997, in the context of the minimal elastic compliance problem, in which the euclidean metric cost was naturally involved. In this talk we will present different variants arising in optimal design of mechanical structures, focusing in particular on the optimization of the pre-stressed elastic sheet. We show that the underlying metric cost is associated with an unknown maximal monotone map which maximizes the Monge-Kantorovich distance between two measures. (Joined work with Karol Bolbotowski, Warsaw)

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**Andrea Braides**

*Concentration problems for scale-invariant energies in periodic media*

I consider the asymptotic description of the minimal-capacity problems for small sets (of given diameter  $\epsilon$ ) at the critical scaling (e.g. the d-capacity in a d-dimensional domain) in a periodically fast-oscillating medium (of period  $\delta$ ). We show that in a sense we always have a separation-of-scale effect: there exists a parameter  $\lambda$  (explicitly computed and depending on the asymptotic mutual behaviour of  $\delta$  and  $\epsilon$  such that at scales less than  $\epsilon^\lambda$  we first have a concentration process around the small set, whose location is subsequently optimized, while for scales larger than  $\epsilon^\lambda$  the concentration process takes place after homogenization. This process gives an explicit formula for the asymptotic capacity. Work with G.C.Brusca.

A similar process occurs at the emergence of topological singularities for a Ginzburg-Landau model of parameter  $\epsilon$  in an inhomogeneous medium of periodicity  $\delta$  as these parameters tend to 0. Since in this case it is a degree condition that forces concentration instead of a Dirichlet boundary condition, the formula for the vortex energy is different. Work in collaboration

with R.Alicandro, M.Cicalese, L.De Luca and A.Piatnitski.

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**Giuseppe Buttazzo**

*Upper and lower bounds for some shape functionals*

The relations between two quantities related to the Laplace operator are considered. In particular, taking as a model the heat diffusion, governed by the heat equation, we aim to study the relations between the average temperature of a heated body and the temperature decay rate of the body in absence of heat sources. The quantities above are expressed by the so-called “torsional rigidity” and by the principal eigenvalue of the Laplace operator. The relations above are studied in the classes of general domains, convex domains, and domains with a small thickness. This allows to obtain a detailed description of the Blasche-Santaló diagram of the two quantities. Several open questions are discussed, in particular when the Laplacian is replaced by the  $p$ -Laplacian.

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**Filippo Cagnetti**

*Rigidity for perimeter and Pólya-Szegő inequalities under spherical symmetrisation*

Spherical symmetrisation operates on a set  $E \subset \mathbb{R}^n$  in the following way. For each given  $r > 0$ , the intersection of  $E$  with the sphere of radius  $r$  centred at the origin is substituted with a spherical cap of the same  $((n - 1)$ -dimensional) measure. This procedure preserves the volume and does not increase the perimeter, so that a perimeter inequality under spherical symmetrisation holds true. We give necessary and sufficient conditions for rigidity of this inequality. That is, we give a characterisation of the situations in which uniqueness of the extremals (up to orthogonal transformations) holds true. After that, we discuss work in progress in which this symmetrisation is applied to Sobolev functions. In this case, one can prove that a Pólya-Szegő inequality holds true (i.e. the symmetrisation acts monotonically on the  $L^p$ -norm of the gradient), and rigidity for this inequality can be studied. This is work in collaboration with Yorgos Domazakis (University of Sussex), Matteo Perugini (University of Milan), Dominik Stöger (Catholic University of Eichstaett-Ingolstadt).

**Riccardo Cristoferi**

*The effect of corners in the relaxation of functionals with bulk and boundary energies*

Boundary effects play an important role in numerous physical and industrial applications. It is thus interesting to understand how they interact with the bulk effects and the geometry of the container. The model case considered here is inspired by the van der Waals–Cahn–Hilliard theory of liquid-liquid phase transitions: the bulk term is the total variation of a scalar function, and the boundary term is given by an integral contact energy. When smooth domains are considered, conditions ensuring lower semi-continuity of the functional have been identified by Modica, and later generalized by Fonseca and Leoni to a broader class of functionals. It is known that these conditions are not sufficient to ensure lower semi-continuity when the domain has corners. In this talk, the relaxation of the above model in the  $L^1$  topology in the presence in a domain with corners is investigated. In particular, the focus is on the effect of the geometry of the container on the relaxation procedure. This talk is based on a joint work with Giovanni Gravina (Temple University).

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**Bernard Dacorogna**

*On the equation  $(\nabla u)^t H \nabla u = G$*

Given two matrices  $G, H : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ , we want to find a map  $u : \mathbb{R}^n \rightarrow \mathbb{R}^n$  such that

$$(\nabla u(x))^t H(u(x)) (\nabla u(x)) = G(x).$$

We discuss the existence, regularity and uniqueness of solutions, as well as boundary conditions. With: S. Bandyopadhyay, V. Matveev and M. Troyanov.

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**Gianni Dal Maso**

*New results on the jerky crack growth in elastoplastic materials*

In the framework of a model for the quasistatic crack growth in elasto-plastic homogeneous materials in the planar case, we study the properties of the length of the crack as a function of time. We prove that, under suitable

technical assumptions on the crack path, this monotone function is a pure jump function. Under stronger assumptions we prove also that the number of jumps is finite.

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**Nicola Fusco**

*The isoperimetric inequality outside a convex set: case of equalities*

In 2007 Choe, Ghomi and Ritoré proved a relative isoperimetric inequality outside a convex set  $C$  together with a characterization of the equality cases for  $C$  smooth. I will present a recent result, proved in collaboration with Massimiliano Morini, where a characterization of the equality case is provided for arbitrary convex sets.

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**Wilfrid Gangbo**

*Global well posedness of a non-local Hamilton-Jacobi equation*

We recall the concepts of common and individual noises in Mean Field Games. Inspired by the theory of optimal mass transportation, we propose a structural condition on Hamiltonians, which we term displacement monotonicity condition, to study second order mean field games master equations on the set of probability measures. A rate of dissipation of a bilinear form is brought to bear a global (in time) well-posedness theory, based on a-priori uniform Lipschitz estimates on the solution in the measure variable.

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**Richard D. James**

*Design of origami structures with curved tiles between the creases*

An efficient way to introduce elastic energy that can bias an origami structure toward desired shapes is to allow curved tiles between the creases. (Think: a Frank Gehry building with creases, that folds up spontaneously from a flat sheet.) Isometric bending of the tiles then supplies the energy. The  $h^3$  scaling of the energy of thin sheets ( $h =$  thickness) spans a broad energy range, that is also consistent with a single origami design. And with a given design, different tiles can have different values of  $h$ . Even a single tile can have differing values of  $h$ . In this lecture we present a theory and systematic design methods for quite general curved-tile origami structures that can be folded

from a flat sheet. Unlike the standard approach to origami design (which is Eulerian), we find it useful to develop Lagrangian methods. A group orbit method using discrete isometry groups enables the design of complex structures from simple calculations. Kirchhoff's nonlinear plate theory is ideal for accurate calculation of the energy. Joint work with Huan Liu, graduate student at UMN.

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**David Kinderlehrer**

*Towards a gradient flow for microstructure*

Cellular networks are ubiquitous in nature. Most engineered materials are polycrystalline microstructures composed of a myriad of small grains separated by grain boundaries, thus comprising cellular networks. A central problem is to develop technologies capable of producing an arrangement, or ordering, of the material, in terms of mesoscopic parameters like geometry and crystallography, appropriate for a given application. Is there such an order in the first place? In this expository talk, we describe the emergence of the grain boundary character distribution (GBCD), an experimentally derived statistic that details texture evolution, and illustrate why it should be considered a material property. We are able to exhibit the harvested statistics as the iterates of an optimal transport JKO implicit scheme, indeed the discrete Euler-Lagrange Equation for iterates. We found this astonishing. This means that the GBCD is the solution of a Fokker-Planck Equation. There is much activity, both in theory and experiment, that we hope to touch on (joint work with P. Bardsley, K. Barmak, E. Eggeling, M. Emelianenko, Y. Epshteyn, X.-Y. Lu and S. Ta'asan).

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**Carolyn Kreisbeck**

*Dealing with nonlocalities in variational functionals: Convexity notions, lower semicontinuity, and relaxation*

Nonlocal variational problems arise in various applications, such as continuum mechanics, the theory of phase transitions, or image processing. Naturally, the presence of nonlocalities leads to new effects, and the standard methods in the calculus of variations, which tend to rely intrinsically on localization arguments, do not apply. In this talk, we address questions arising

from the existence theory for three different classes of variational functionals: integrals depending on Riesz fractional gradients, double integrals, and double supremals - and find qualitatively very different results. Regarding the characterization of weak lower semicontinuity, it may be surprising that quasiconvexity, which is well-known from the classical local setting, provides the correct convexity notion also for the fractional integrals. Our proof relies on a translation mechanism that allows switching between classical and fractional gradients. In the case of double supremals, we show that the natural guess of separate level convexity fails in the vectorial case and introduce the new Cartesian level convexity. As for relaxation, we discuss the central issue of why these nonlocal functionals, in contrast to their local counterparts, cannot be expected to preserve their structure when taking weak lower semicontinuous envelopes. This is based on joint work with and Antonella Ritorto, Hidde Schönberger (both KU Eichstätt-Ingolstadt) and Elvira Zappale (Sapienza University of Rome).

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**Stefan Krömer**

*Injective elastic deformations via vanishing self-repulsion*

For many models of elastic solids, injectivity of deformations is a crucial constraint because it represents non-interpenetration of matter. On the theoretical level, this is usually achieved by imposing the well-known Ciarlet-Nečas condition. Beyond mere heuristics, its approximation, especially mathematically rigorous and computationally efficient approximation, is much less well understood, though. I will present recent theoretical and practical progress in this area, using self-repulsion terms concentrated on or near the domain boundary. These penalize self-interpenetration, possibly even preventing it completely for finite energy, while correctly reproducing the injectivity constraint in a limiting sense. For practical numerical tests, we also consider a linear elastic model augmented with locking-type constraints to ensure sufficient local invertibility. This is joint work with Jan Valdman (UTIA Prague) and Philipp Reiter (TU Chemnitz).

**Martin Kružík***Hadamard's inequality in the mean*

Let  $Q$  be a Lipschitz domain in  $\mathbb{R}^2$  and let  $f \in L^\infty(Q)$ . We investigate conditions under which the functional

$$I(\varphi) = \int_Q |\nabla\varphi|^2 + f(x) \det \nabla\varphi \, dx$$

obeys  $I(\varphi) \geq 0$  for all  $\varphi \in W_0^{1,2}(Q, \mathbb{R}^2)$ . We prove that there are  $f$  such that  $I \geq 0$  holds and is strictly stronger than the best possible inequality that can be derived using Hadamard's pointwise inequality  $2|\det A| \leq |A|^2$  alone. Almost all of the  $f$  we consider are piecewise continuous, and we find that in these cases it is both the geometry of the 'jump sets' as well as the sizes of the 'jumps' themselves that determine whether  $I \geq 0$  holds. We also outline connections to quasiconvexity at the boundary and to Agmon's conditions. Theoretical results will be complemented with various numerical experiments. This is a joint work with J. Bevan (Surrey) and J. Valdman (Prague).

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**Mitchell Luskin***Mathematics and Physics at the Moiré Scale and the Calculus of Variations*

Placing a two-dimensional lattice on another with a small rotation gives rise to periodic "moiré" patterns on a superlattice scale much larger than the original lattice. This effective large-scale fundamental domain allows phenomena such as the fractal Hofstadter butterfly in the spectrum of Harper's equation to be observed in real crystals. Experimentalists have more recently observed new correlated phases at the "magic" twist angles predicted by theorists. The accurate modeling of the relaxation of twisted bilayer 2D materials is essential to the prediction of its electronic properties. We will derive an elastic energy for 2D heterostructures and present analysis and numerical analysis for its minimization. We will also give mathematical and computational models to predict and gain insight into new physical phenomena at the moiré scale including our recent mathematical and experimental results for twisted trilayer graphene.

## Paolo Marcellini

*Regularity for non-uniformly elliptic equations and systems*

We give some interior and boundary regularity results for weak solutions to some non-uniformly elliptic equations and systems, as well as some examples and remarks about cases with possibly not regular weak solutions. Part of the material of this talk is based on joint works recently published in the following papers:

- V. Bögelein, F. Duzaar, P. Marcellini, C. Scheven: Boundary regularity for elliptic systems with  $p, q$ -growth, *J. Math. Pures Appl.*, 159 (2022), 250–293.  
<https://doi.org/10.1016/j.matpur.2021.12.004>
- G. Cupini, P. Marcellini, E. Mascolo, A. Passarelli di Napoli: Lipschitz regularity for degenerate elliptic integrals with  $p, q$ -growth, *Advances in Calculus of Variations*, (2021). In press.  
<https://doi.org/10.1515/acv-2020-0120>
- M. Eleuteri, P. Marcellini, E. Mascolo, S. Perrotta: Local Lipschitz continuity for energy integrals with slow growth, *Ann. Mat. Pura Appl.*, 4 (2021).  
In press. <https://doi.org/10.1007/s10231-021-01147-w>

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## Vincent Millot

*Torus and split solutions of the Landau-de Gennes model for nematic liquid crystals*

In this talk, I will present the  $Q$ -tensor model of Landau-de Gennes for nematic liquid crystals in the so called Lyutsyukov regime dealing with maps with values in the 4-dimensional sphere. This model describes stable configurations of a liquid crystal as minimizers of a Ginzburg-Landau type energy in which the potential well is the real projective plane, seen as a submanifold of  $S^4$ . In the case where the 3D domain is the unit ball and the Dirichlet boundary data is radially symmetric (equivariantly), one may expect that a minimizer inherits such symmetry. Classical simulations show that this is not the case and a certain toroidal structure appears. If (equivariant) axial

symmetry is imposed to reduce the complexity of the problem, another type of “singular” solutions appears, the split solutions. By means of regularity results on this model, I will discuss the existence / geometry of torus and split solutions and explain the strong dependence of the type of solutions with respect to the boundary condition and the shape of the domain. This talk is based on recent works in collaboration with Federico Dipasquale and Adriano Pisante.

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**Maria Giovanna Mora**

*The energy of a Möbius strip and other ribbons*

It is observed that a Möbius strip made of an unstretchable material, when left to itself, adopts a characteristic equilibrium shape, which is independent of the material the strip is made of (if sufficiently stiff to ignore gravity). To the best of our knowledge, Michael Sadowsky in 1930 was the first to address the problem of finding this “universal shape”. He formulated the problem variationally in terms of an energy obtained as a formal limit of the Kirchhoff energy for a band of vanishing width. This energy (now known as the Sadowsky energy) depends on the curvature and torsion of the centerline of the band and it is singular at points with zero curvature. In this talk we will re-examine the derivation of the Sadowsky energy using Gamma-convergence, also including boundary conditions, and we will discuss how this relates to the Kirchhoff energy for plates and to 3d nonlinear elasticity.

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**Massimiliano Morini**

*Capillary surfaces and a model of nanowire growth*

After recalling the classical variational formulation of the capillarity problem and some related results, we consider a model for vapor-liquid-solid growth of nanowires proposed in the physical literature. In this model, liquid drops are described as local or global volume-constrained minimizers of the capillarity energy outside a semi-infinite convex obstacle modeling the nanowire. We first address the existence of global minimizers and then, in the case of rotationally symmetric nanowires, we investigate how the presence of a sharp edge affects the shape of local minimizers and the validity of Young’s law. Finally, we study the regularity of the contact line between the drop and the

nanowire near the sharp edge.

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**Felix Otto**

*Universality of the magnetization ripple: A singular SPDE-perspective*

The ripple in a thin-film ferromagnet is the magnetization's anisotropic response to poly-crystallinity; it is experimentally well-documented. The randomly oriented grains come with a favorite axis for the magnetization that acts as a random field, a source of quenched noise.

On a mesoscopic level, the ripple is modeled by a 2-d anisotropic and non-local variational problem with a random field term converging to white noise. White noise itself is too rough for the Burgers' type nonlinearity to have a classical sense, so that a renormalization is called for. We follow the strategy of rough path/regularity structures in the sense that we enhance the noise description.

The main result is that of universality: The mesoscopic ensemble of the ripple is independent on *how* the limit of white noise is approached, it is thus independent of the statistics of the grain arrangement. In view of the non-convexity of the variational problem, this statement is formulated in terms of a suitable  $\Gamma$ -topology.

On the technical side, we appeal to the spectral gap inequality to avoid the assumption of Gaussianity of the approximation. We also leverage a fluid-type identity for the Burgers equation to obtain coercivity of the renormalized variational problem.

This is joint work with Radu Ignat, Tobias Ried, and Pavlos Tsatsoulis.

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**Roberto Paroni**

*Microscopically piece-wise rigid plates inspired by graphene*

In this talk we present an atomistic to continuum model for a graphene sheet undergoing bending, within the small displacements approximation framework. Under the assumption that the atomic interactions are governed by a harmonic approximation of the 2nd-generation Brenner REBO (reactive empirical bond-order) potential, we determine the variational limit of the

energy functionals. If some specific contributions in the atomic interaction are neglected, the variational limit is non-local. We then analyze the results and by making a connection with the classical theory of plates we will be lead to introduce a new material property: the bending Poisson coefficient. Finally, we consider some extreme cases of our model and this will bring us to microscopically piece-wise rigid plates. The talk is based on joint works with C. Davini, A. Favata, and A. Micheletti.

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**Pablo Pedregal**

*Optimal control governed by hyper-elasticity*

The study of optimal control of non-linear PDEs almost reduces to the semi-linear case, and for single equations. In this talk, and motivated by applications, we will describe several problems in which the set of governing state equations is the non-linear Euler-Lagrange system coming from hyper-elasticity. The variational structure of the state system makes possible to treat such sophisticated situations, at least in some simple cases. Though the analytical proofs are not hard, as they are based on polyconvexity and the weak continuity of minors, the range of applications is quite appealing. We will focus on three particular situations, and show several pictures/movies resulting from the numerical approximation.

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**Paolo Piovano**

*Microscopic justification of the Winterbottom shape*

The continuum model related to the Winterbottom problem, namely the problem of determining the equilibrium shape of crystalline drops resting on a half-plane substrate, is derived by means of a rigorous discrete-to-continuum passage performed by Gamma-convergence from atomistic models. Such atomistic models are introduced by considering (non-interpenetrating) reference lattices for the substrate and the drop particles, by fixing all the substrate particles, and by letting the drop particles instead free to interact both among themselves and with the substrate particles through different atomistic potentials, which are chosen of Heitmann-Radin type. The various positionings of the reference lattices are treated by singling out a few energetically equivalent settings to which the analysis is reduced.

The investigations generalize previous results available in the literature for the Wulff shape to the presence of a substrate wall and, as a byproduct of the analysis, effective expressions for the anisotropy at the free drop boundary and the drop wettability at the substrate wall are characterized in terms of the chosen atomistic potentials. A threshold condition only depending on such potentials is determined distinguishing the wetting regime, where discrete minimizers are explicitly characterized as configurations contained in a one-atom thick layer on the substrate, from the dewetting regime. In the latter regime, also in view of a proven conservation of mass in the limit as the number of drop particles tends to infinity, proper scalings of the discrete minimizers converge to a bounded minimizer of the Winterbottom continuum model satisfying a nonzero volume constraint.

The presented results are outcomes of a collaboration with Igor Velčić (Zagreb, Croatia).

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**Dejan Slepčev**

*Nonlocal Wasserstein metric: properties, asymptotics, and gradient flows*

The seminal result of Benamou and Brenier provides a dynamical description of the Wasserstein distance as the path of the minimal action in the space of probability measures, where paths are solutions of the continuity equation and the action is the kinetic energy. We consider a fundamental modification of the framework where the paths are solutions of nonlocal continuity equations and the action is a nonlocal kinetic energy. The resulting nonlocal Wasserstein distances are relevant to fractional diffusions and Wasserstein distances on graphs. We will present the basic properties of the distance and conditions on the (jump) kernel that determine whether the topology metrized is the weak or the strong topology. We will also discuss quantitative comparisons between the nonlocal and local Wasserstein distance. Finally we will discuss gradient flows with respect to the nonlocal Wasserstein metric.

**Konstantina Trivisa**

*An efficient quantum algorithm for dissipative nonlinear partial differential equations*

Models governed by both ordinary differential equations and partial differential equations arise extensively in the natural and social sciences, medicine, and engineering. Such equations characterize physical and biological systems that exhibit a wide variety of complex phenomena, including turbulence and chaos. In this talk, we focus on differential equations with nonlinearities that can be expressed with quadratic polynomials, which include many archetypal models in biology, fluid dynamics, and plasma physics. Quantum algorithms offer the prospect of rapidly characterizing the solutions of high-dimensional systems. Such algorithms can produce a quantum state proportional to the solution of a sparse (or block- encoded)  $n$ -dimensional system of linear differential equations in time  $\text{poly}(\log n)$  using the quantum linear system algorithm (QLSA) (cf. Harrow et al (2009), Childs et al (2016)). Whereas previous quantum algorithms for general nonlinear differential equations have complexity exponential in the evolution time, our work gives the first quantum algorithm for dissipative nonlinear differential equations that is efficient provided the dissipation is sufficiently strong relative to nonlinear and forcing terms and the solution does not decay too rapidly. We also establish a lower bound showing that differential equations with sufficiently weak dissipation have worst-case complexity exponential in time, giving an almost tight classification of the quantum complexity of simulating nonlinear dynamics. Furthermore, numerical results for the Burgers equation suggest that our algorithm may potentially address complex nonlinear phenomena even in regimes with weaker dissipation. Finally, we discuss potential applications, showing that the imposed condition can be satisfied in realistic epidemiological models. The article “Efficient quantum algorithm for dissipative nonlinear partial differential equations” appeared recently in the Proceedings of the National Academy of Sciences (PNAS 2021).

Authors: J-P Liu, H. Kolden, H. Krovi, N. Loureiro, K. Trivisa and A. Childs.

**Raghavendra Venkatraman**

*Asymptotic Expansions for the Spectrum of Periodic Schrodinger Operators.*

We present recent results on quantitative homogenization of the spectrum of Schrodinger operators in periodic media in  $R^d$  with a confining potential. Joint work with Scott Armstrong (NYU).

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**Elvira Zappale**

*Variational formulation for hierarchies of structured deformations*

The mechanical theory of first order structured deformations formulated in [2] enriches the purely macroscopic field theory of non-linear elasticity by taking into account the effects of disarrangements that occur at a single submacroscopic level. In [1] a variational formulation for this theory has been successfully provided in the context of special fields of bounded variation.

Taking into account that many natural and man-made physical systems have a rich enough geometrical structure to permit the identification of hierarchies consisting of more than one physically meaningful submacroscopic level, the mechanical theory of structured deformations has been enriched in [3]. This talk aims at presenting a variational framework of these latter theoretical settings.

References:

1. R. Choksi and I. Fonseca: Bulk and interfacial energy densities for structured deformations of continua. Arch. Rational Mech. Anal. 138 (1997), 37-103.
2. G. Del Piero and D. R. Owen: Structured deformations of continua: Arch. Ration. Mech. Analysis -124 (1993), 99-155.
3. L. Deseri and D. R. Owen: Elasticity with Hierarchical Disarrangements: A Field Theory That Admits Slips and Separations at Multiple Submacroscopic Levels: J.of Elasticity Vol- 135 (2019) Issue 1-2, 149-180.

Joint project with Ana Cristina Barroso, José Matias, Marco Morandotti & David Owen.

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**Barbara Zwicknagl**  
**TBA**